The string tension in the maximally Abelian gauge after smoothing.

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We apply smoothing to SU(2) lattice field configurations in 3+1 dimensions before fixing to the maximally Abelian gauge. The Abelian projected string tension is shown to be stable under this, whilst the monopole string tension declines by $\mathcal{O}(30\%)$. Blocking of the SU(2) fields reduces this effect, but the use of extended monopole definitions does not. We discuss these results in the context of additional confining excitations in the U(1) vacuum.

In recent years a confinement mechanism based on a dual superconducting vacuum [1,2] has been studied, motivated in large part by the observation that in SU(2) in the maximally Abelian (MA) gauge [3], degrees of freedom (d.o.f.) may be numerically integrated out ('Abelian projection') to yield U(1) configurations reproducing the original string tension, and that magnetic monopoles are responsible for this [4–6]. A physically attractive scenario stems from the assumption that the non–Abelian theory may be written as a combination of UV and IR d.o.f. All the IR physics is postulated to remain in the residual U(1) fields after fixing to the MA gauge. The full SU(2) string tension should then be calculable from an Abelian Wilson loop in this gauge.

Application of short range perturbations to the SU(2) fields cannot alter the asymptotic string tension, nor is it expected to change any estimate of this from a correlation function extending over a scale much greater than the range of the perturbation. A crucial test of the above scenario is that the U(1) string tension after gauge fixing behaves similarly, and that Abelian dominance remains.

Smoothing of the SU(2) fields is such a perturbation. We generate an ensemble of SU(2) field configurations using the Wilson plaquette action at $\beta = 2.5$ on 16^4, 20^4; lattices large enough that we may estimate the asymptotic string tension. To this ensemble is applied one Wilson action smoothing (= cooling) step. Each link in ('staggered') turn on the lattice is updated to locally minimise the Wilson action. This is a local smoothing on the scale of one lattice spacing and will not affect the string tension which we extract on a scale of several lattice spacings [7].

In this work smoothed configurations are fixed to the MA gauge and Abelian projected to create an ensemble of U(1) fields and the magnetic monopole worldlines are identified as in [8]. We define effective U(1) and monopole string tensions from the square Creutz ratios: $K_{\text{eff}}(r) = -\ln C(r, r)$, which in the limit of large r converge to the asymptotic string tensions, $K = a^2\sigma$.

The U(1) effective string tension converges after one smoothing sweep to the asymptotic SU(2) value before smoothing, but with reduced statistical errors (Fig 1). The smoothing is thus only a local U(1) perturbation, in support of the aforementioned scenario. Interestingly, as at $\beta = 2.4$ [9], the monopole string tension is suppressed by $\mathcal{O}(30\%)$ by the single smoothing sweep. This implies that, in addition to magnetic monopoles, there are other objects present in these smoothed U(1) fields that disorder Wilson loops and contribute to the string tension. (Since their magnetic flux is conserved these objects may be thought of as 'vortices'.) This is not a complete surprise. If we have Abelian dominance (in the sense of U(1) and SU(2) plaquettes
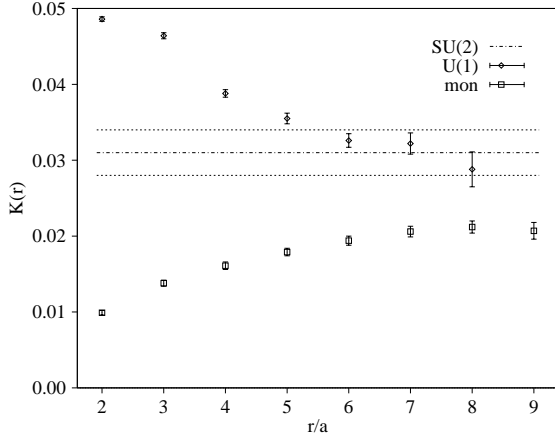
Figure 1. The U(1) and monopole effective string tensions after 1 smooth on 16^4.

Figure 2. The monopole effective string tension after blocking on 20^4.

being similar) then smoothed SU(2) fields will produce smooth U(1) fields which cannot contain singular monopole cores.

Blocking an SU(2) configuration creates a new lattice of $(L/2)^4$ sites with links formed by summing over staples:

$$V_\mu(n') = D_\mu(n) + \sum_\nu S^f_{\mu\nu}(n) + S^b_{\mu\nu}(n)$$

where $D_\mu(n) \equiv U_\mu(n).U_\mu(n + \hat{\mu})$, $S^f_{\mu\nu}(n) = U_\nu(n).D_\mu(n + \hat{\nu}).U^\dagger_\nu(n + 2\hat{\mu})$, $S^b_{\mu\nu}(n) = U^\dagger_\nu(n - \hat{\nu}).D_\mu(n - \hat{\nu}).U_\nu(n - \hat{\nu} + 2\hat{\mu})$ and V is projected back into the group. The long range physics of such a lattice will be the same, up to a doubling of the lattice spacing (e.g. the asymptotic SU(2) string tension, K, will be four times that of the unblocked lattice, as is plotted here). An ensemble of (unsmoothed) SU(2) configurations, blocked and then fixed to the MA gauge should thus show Abelian and monopole dominance of the asymptotic string tension. This is so; in Fig. 2 we illustrate the latter, and as before blocking $K_{\text{eff}}(r)$ assumes its asymptotic behaviour for small r.

One smoothing sweep is sufficiently localised that the difference between the smoothed and unsmoothed configuration should be greatly reduced after blocking. We thus expect at least partial restoration of the monopole dominance that was lost under smoothing. In Fig. 2 we compare the
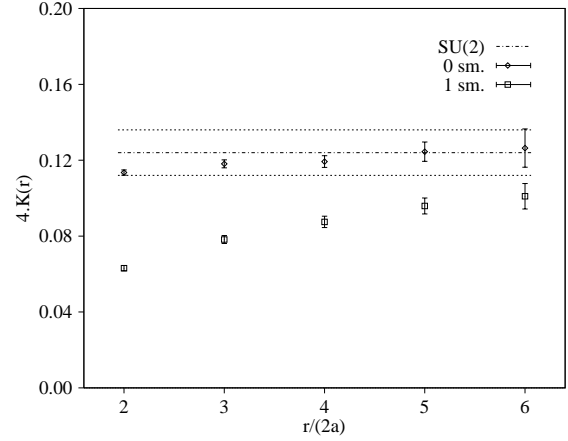
smoothed and unsmoothed case for a blocked 20^4 lattice. Although there is not a clear plateau in the effective string tension from monopoles after blocking, in comparison to Fig. 1 the loss of monopole dominance is much reduced.

We investigated whether the effect of blocking the SU(2) links could be obtained by working with 'extended' monopoles [10]. The U(1) plaquette angles are concentrated around multiples of 2π and interpreted as Dirac strings. The net number of these entering an elementary cube gives its monopole charge. Blocking these charges yields 'Type-II' extended monopoles; we sum the elementary charges in a 2^3 block. Whilst this eliminates some magnetic dipoles, it is clear that there can be no new physics and we employ this as a control. 'Type-I' extended monopoles are found by applying the above definition to 2×2 Wilson loops, and the monopole charges assigned to 2^3 blocks correspond to the elementary monopoles seen after a (staple-less) blocking of the U(1) fields. If using Type-I monopoles is to reproduce blocking the SU(2) links, Type-I monopoles should give a higher string tension than Type-II. They are however very alike (Fig. 3). Although similar to that of the blocked lattices at small distances, ultimately both Types-I and II appear to form a plateau well below the SU(2) string tension. In Fig. 4, we go further to compare the microscopic properties of Types-I and II by lo-
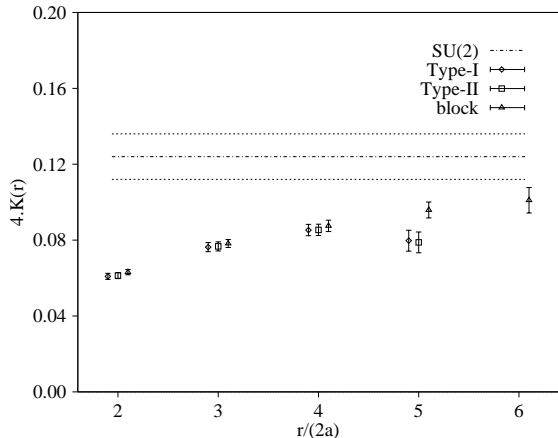
Figure 3. The monopole string tensions after 1 smooth (extended on 16^4, blocking on 20^4).

Figure 4. The effective string tension for the extended monopole difference gas on 16^4.

cally forming a 'difference gas' on each configuration. Before smoothing, there are some differences between the definitions, but this is only short ranged. After smoothing, when we would be most interested in Type-I identifying new magnetic excitations, we see that they are identical at all length scales.

In conclusion, we have applied smoothing to SU(2) gauge configurations and found no reduction in the U(1) string tension after fixing to the MA gauge. This is consistent with the scenario in which the MA gauge isolates the long range d.o.f. in the SU(2) theory. The monopole string tension, however, has a marked decline, $\mathcal{O}(30\%)$ at $\beta = 2.5$. This suggests the presence of additional objects in the U(1) fields capable of producing confinement, and also their increased rôle in the smoothed fields. These did not resemble extended monopoles, which failed to restore the monopole string tension. Applying a blocking transformation to the smoothed SU(2) fields prior to gauge fixing led to an approximate re–emergence of the monopole dominance. Previous studies here indicate a sensitivity to the smoothing method; Metropolis cooling of SU(2) is mild enough not to destroy the monopole string tension [11], but a renormalisation group based smoothing algorithm reduces even the U(1) string tension [12]. There is here a complex interplay between elementary monopoles and other U(1) objects under
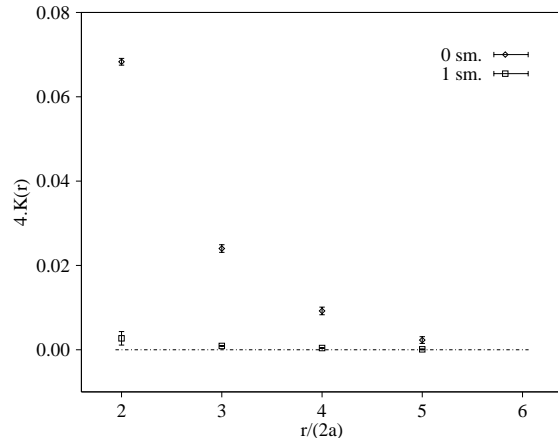
smoothing, and this and the effects of blocking merit further study.

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